Unified Engineering / Thermodynamics

Thermal radiation problem

Consider a spacecraft of mass M and specific heat c traveling at an altitude of 500 km above Earth's surface. The vehicle is completely covered by a multi-layer thermal shield, except for the surface exposed towards the Earth (the radiator). Assume the shield acts as an adiabatic enclosure, so no heat is emitted or absorbed by it.
Write down a heat power equation that would describe the temperature evolution in time of the spacecraft. Assume electronic components inside the spacecraft create a heat dissipation load $P_D = 100$ W. Assume the Earth's albedo (fraction of solar radiation reflected by the Earth) is $a = 0.3$. Take the Earth's surface temperature at 300°K.
After a long time, the temperature of the spacecraft will stop changing. A steady state equilibrium situation will be reached. **Calculate the equilibrium temperature**.

For the radiator assume:

Emissivity: $\varepsilon = 0.1$ (for instance, oxidized aluminum)
Absorption: $\alpha = \varepsilon = 0.1$ (Kirchoff's law)
Area: $A_r = 1\ \text{m}^2$

Surface temperature of the Sun: 5800°K
Radius of the Sun: 695,000 km
Earth-Sun distance: 149,597,892 km
Radius of the Earth: 6379 km